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Building on recent work by Brodsky *et al.*, we advocate searching for glueball degrees of freedom in $e^+e^- \rightarrow J/\psi \rightarrow \phi f_0$ at CLEO-c and BES.

Brodsky, Goldhaber, and Lee [1] have proposed a novel approach to producing (scalar) glueballs in e^+e^- annihilation to account for the anomalously large cross sections for $J/\psi + \eta_c$, χ_{c0} , and $\eta_c(2S)$ observed at Belle [2]. They made a pQCD estimate of the cross section for $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi \mathcal{G}_0$ at $\sqrt{s} = 10.6$ GeV, and found it to be similar to the exclusive charmonium-pair production $e^+e^- \rightarrow J/\psi h$ for $h = \eta_c$ and χ_{c0} . Further, since $\gamma^* \rightarrow (c\bar{c})(c\bar{c})$ and $\gamma^* \rightarrow (c\bar{c})(gg)$ were of the same nominal order, they suggested that some portion of the anomalously large signal observed by Belle in $e^+e^- \rightarrow J/\psi X$ may actually be due to the production of $J/\psi \mathcal{G}_0$.

This is an interesting idea theoretically but has a potential limitation phenomenologically. As presented, the work of Ref. [1] applies when $M_{\mathcal{G}} \simeq M_{J/\psi} \simeq 3$ GeV. However, Lattice QCD [3, 4] and phenomenological studies [5, 6] suggest a much smaller mass scale for the lightest scalar glueball $M_{\mathcal{G}} \simeq 1.5$ GeV. Some analyses suggest even lower glueball masses [7, 8]. Taking into account these factors, we anticipate that the mass scale for the lightest scalar glueball is smaller than 3 GeV.

Thus we consider here the application of the work of Ref. [1] to the scenario of a scalar glueball in the $O(1)$ GeV mass region. The analysis of Ref. [1] allows one to rescale the kinematics such that instead of a 3-GeV glueball recoiling against a J/ψ , we may consider a 1-GeV glueball recoiling against a ϕ . Also, rescaling the c.m. energy by a factor of three brings one to the kinematic region of interest currently at BES and to CLEO-c.

In Ref. [1], the mass scale is introduced via the mass ratio $r = 4m_c/\sqrt{s}$, where $m_c = M_{J/\psi}/2$ is the charm quark mass. By choosing the glueball mass $M_{\mathcal{G}} = M_{J/\psi}$, the phase space factor for $J/\psi \mathcal{G}_0$ production cancels in the branching ratio fraction of $J/\psi \mathcal{G}_0$ to $J/\psi \eta_c$. As a result, the s -dependence of the branching ratio fraction will be embedded in r apart from strong couplings and nonperturbative factors determined through the quarkonium decay in its rest frame. Due to this feature, given that $M_{\mathcal{G}} = M_{(q\bar{q})}$, the branching ratio fraction of $q_h \bar{q}_h \rightarrow \gamma^* \rightarrow (q\bar{q}) \mathcal{G}_0$ to $q_h \bar{q}_h \rightarrow \gamma^* \rightarrow (q\bar{q})(q\bar{q})$ scales in terms of r apart from a constant, where q_h denotes a heavy quark.

First, we examine the process $\gamma^* \rightarrow \phi(gg)$ in parallel to $\gamma^* \rightarrow J/\psi(gg)$. An important argument of Ref. [1] is that the decays of $\gamma^* \rightarrow (q\bar{q})(gg)$ and $\gamma^* \rightarrow (q\bar{q})(q\bar{q})$ are the same order (see Fig. 1(a)-(b)). The ratio of $\gamma^* \rightarrow \phi \mathcal{G}_0$ to $\gamma^* \rightarrow \mu^+ \mu^-$ can be estimated by applying Eq. (7) of Ref. [1]:

$$\frac{R_{\phi \mathcal{G}_0}}{R_{\mu^+ \mu^-}} = \frac{32\pi^2 \alpha_s^2 e_s^2 r^2 (1 + r^2/2) \Phi_0^{ee} \langle O_1 \rangle_\phi |I_0|^2}{9(1 - r^2/4)^2 m_s^3 s}, \quad (1)$$

where e_s and $m_s = M_\phi/2$ are the s quark's charge and mass. The gluon distribution factor $|I_0|^2$ was assumed to be a function of the glueball's J^{PC} and to scale with mass, so we adopt the same form as Ref. [1]. Φ_0^{ee} is a phase space factor: $\Phi_0^{ee} = \sqrt{[1 - (M_\phi + M_{\mathcal{G}})^2/s][1 - (M_\phi - M_{\mathcal{G}})^2/s]}$. Under the condition of $M_{\mathcal{G}} = M_\phi$ and $m_s = M_\phi/2$, we have $\Phi_0^{ee} = \sqrt{1 - r^2}$. For these reduced energies we adopt the running coupling constant $\alpha_s \sim 0.33$ at $\sqrt{s} = 3.1$ GeV as a guide, and assume also $\alpha_s^{\mathcal{G}} = \alpha_s^{\eta'} = 0.33$ in analogy with the treatment of Ref. [1]. The matrix element $\langle O_1 \rangle_\phi$ is given by the radial wavefunction of the $s\bar{s}$ in the ϕ at the origin $R(0)$ by analogy with the case of $c\bar{c}$: $\langle O_1 \rangle_\phi = |R(0)|^2 N_c/2\pi = 2M_\phi f_\phi^2$, where f_ϕ is the decay constant of the ϕ meson.

For a glueball mass $M_{\mathcal{G}} \simeq 1$ to 1.7 GeV, by analogy with Ref. [1], we would compare $\phi \mathcal{G}$ with $\phi \eta'$ or any of $\phi f_0(980)$, $\phi f_0(1370)$, $\phi f_0(1500)$, $\phi f_0(1700)$, which would be clear if $\eta' = \eta(s\bar{s})$ and $f_0 = f_0(s\bar{s})$. However, in practice, the probability of $s\bar{s}$ in η' is about 1/2. The scalars are even non-trivial. The $f_0(980)$ may be a $K\bar{K}$ molecule, or a $q^2 \bar{q}^2$ state [9, 10]. In either picture it is not simply related to the $s\bar{s}$ content of interest to us. The $f_0(1370)$, (1500), (1700) are believed to be mixtures of \mathcal{G}_0 , $s\bar{s}$ and $n\bar{n}$, so it is not possible to normalize the $\phi \mathcal{G}_0$ to these in a meaningful way [13]. Thus we compare $\phi \mathcal{G}_0$ to $\phi(s\bar{s})$, where $(s\bar{s})$ is an effective ideal $s\bar{s}$ state with the same mass as η' .

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To proceed, the rescaling feature of Eq. (1) (i.e. Eq. (7) of Ref. [1]) should be examined. In Fig. 2, we present the calculations of the branching ratio $R_{\phi\mathcal{G}_0}$ and $R_{J/\psi\mathcal{G}_0}$ in terms of r to show the rescaling features between the ϕ -glueball and J/ψ -glueball production in quarkonium decay via virtual photons. The quantity r is in the range of $0 < r < 1$, which corresponds to the physical region $\sqrt{s} > 4m_q$. For the ideal condition that the phase space factor is cancelled out, the rescaling feature is shown by the constant fraction (dotted curve in Fig. 2(a)) between the J/ψ -glueball and ϕ -glueball production ratios. The ratio reflects the difference of the factors $m_c|I_0|^2/\langle O_1 \rangle_{\eta_c}$ and $m_s|I_0|^2/\langle O_1 \rangle_\phi$, which denote the ratios of the square of the glueball wavefunctions at their origins compared to these of the produced quarkonia. Note that in these two cases the kinematics in terms of r are quite similar as indicated by the arrows. In Fig. 2(b), we also present the calculation including the contributions from the non-cancelling phase space factors with $m_c = 1.4$ GeV and $M_G = M_{\eta_c}$. The phase space factor causes deviations from the exact rescaling between the solid and dashed curves as $r \rightarrow 1$, but is negligible in most of the kinematical regions.

For glueball mass $M_G = M_{\eta'}$, Eq. (1) gives $R_{\phi\mathcal{G}_0}/R_{\mu^+\mu^-} = 9.95 \times 10^{-5}(\alpha_s/0.33)^2$. In association with Eq. (8) of Ref. [1], with $R_{\mu^+\mu^-} = 5.88\%$ [11] we obtain:

$$br_{J/\psi \rightarrow \gamma^* \rightarrow \phi\mathcal{G}} \simeq br_{J/\psi \rightarrow \gamma^* \rightarrow \phi(s\bar{s})} = 5.85 \times 10^{-6}(\alpha_s/0.33)^2 \quad (2)$$

for the production via virtual photons. Thus the work of Ref. [1] provides a method for estimating the virtual photon transitions in $J/\psi \rightarrow \phi(s\bar{s})$, by which the glueball production can be normalized. We now examine the consequence of this estimate, and investigate its prediction for the glueball production.

Apart from the EM transition, the other important process in $J/\psi \rightarrow \phi(s\bar{s})$ is via intermediate gluons, i.e. $J/\psi \rightarrow 3g \rightarrow \phi\eta'(s\bar{s})$. We can thus express the ratio between the EM decay and strong decay of J/ψ as:

$$\frac{br_{J/\psi \rightarrow 3g \rightarrow \phi(s\bar{s})}}{br_{J/\psi \rightarrow \gamma^* \rightarrow \phi(s\bar{s})}} = \frac{br_{J/\psi \rightarrow 3g}}{br_{J/\psi \rightarrow \gamma^*}} \frac{br_{\phi\eta' \rightarrow 3g}}{br_{\phi\eta' \rightarrow \gamma^*}}. \quad (3)$$

For an ideal flavor singlet \mathcal{F} , the ratio for its coupling to gluons and a virtual photon γ^* can be written as

$$\frac{br_{3g \rightarrow \mathcal{F}}}{br_{\gamma^* \rightarrow \mathcal{F}}} \sim \frac{\sigma_{\mathcal{F}}}{e_{\mathcal{F}}^2}, \quad (4)$$

where $\sigma_{\mathcal{F}}$ summarises the flavor dependence of the gluon coupling to the final state configuration, and $e_{\mathcal{F}}$ is the charge factor of the quarks. For the ratio of gluon and photon coupling to the initial J/ψ and $s\bar{s}$, we then have

$$\frac{br_{J/\psi \rightarrow 3g}}{br_{J/\psi \rightarrow \gamma^*}} \frac{br_{\gamma^* \rightarrow s\bar{s}}}{br_{3g \rightarrow s\bar{s}}} = \frac{\sigma_{J/\psi}}{e_c^2} \frac{e_s^2}{\sigma_{s\bar{s}}} \simeq \frac{e_s^2}{e_c^2} = \frac{1}{4}, \quad (5)$$

where we have assumed flavor independence of the quark-gluon coupling. With the experimental values, $br_{J/\psi \rightarrow 3g} = 0.877 \pm 0.005$ and $br_{J/\psi \rightarrow \gamma^*} = 0.17 \pm 0.02$ [11], we have

$$\frac{br_{3g \rightarrow s\bar{s}}}{br_{\gamma^* \rightarrow s\bar{s}}} = 4 \times \frac{br_{J/\psi \rightarrow 3g}}{br_{J/\psi \rightarrow \gamma^*}} = 21 \pm 3. \quad (6)$$

Consequently, we can estimate

$$br_{J/\psi \rightarrow 3g \rightarrow \phi(s\bar{s})} = br_{J/\psi \rightarrow \gamma^* \rightarrow \phi(s\bar{s})} \times \frac{br_{J/\psi \rightarrow 3g}}{br_{J/\psi \rightarrow \gamma^*}} \frac{br_{3g \rightarrow s\bar{s}}}{br_{\gamma^* \rightarrow s\bar{s}}} = (6.5 \pm 1.7) \times 10^{-4}(\alpha_s/0.33)^2, \quad (7)$$

which suggests that

$$br_{J/\psi \rightarrow \phi(s\bar{s})}^{th} = br_{J/\psi \rightarrow 3g \rightarrow \phi(s\bar{s})} + br_{J/\psi \rightarrow \gamma^* \rightarrow \phi(s\bar{s})} = 6.5 \times 10^{-4}(\alpha_s/0.33)^2. \quad (8)$$

In reality, a pure $s\bar{s}$ state with $J^{PC} = 0^{++}$ does not exist: the physical scalar states involve mixing of $s\bar{s}$ with the non-strange $u\bar{u}$ and $d\bar{d}$. We thus compare $br_{J/\psi \rightarrow 3g \rightarrow \phi(s\bar{s})}$ with $br_{J/\psi \rightarrow \phi\eta}$ and $br_{J/\psi \rightarrow \phi\eta'}$ at the J/ψ mass. Taking into account the phase space factor, we estimate the $s\bar{s}$ branching ratio as

$$br_{J/\psi \rightarrow \phi(s\bar{s})}^{exp} \simeq br_{J/\psi \rightarrow \phi\eta}^{exp} \left(\frac{p_{\eta'}}{p_{\eta}} \right)^3 + br_{J/\psi \rightarrow \phi\eta'}^{exp} = (8 \pm 1) \times 10^{-4}. \quad (9)$$

If we neglect the phase space factor, the ratio will be $br_{J/\psi \rightarrow \phi(s\bar{s})}^{exp} \simeq (9.8 \pm 1.1) \times 10^{-4}$ [11], which suggests that phase space is not a significant factor in this estimate. This comparison shows that Eq. (8) is in good agreement with the

experimental data and thereby supports the method of Ref. [1]. In particular, it provides a way to normalize glueball production in J/ψ decays.

The above estimate can be applied to $J/\psi \rightarrow 3g \rightarrow \phi\mathcal{G}_0$ [Fig. 1 (d)], which analogous to Eqs. (7) and (8) gives

$$br_{J/\psi \rightarrow \phi\mathcal{G}_0}^{th} = br_{J/\psi \rightarrow 3g \rightarrow \phi\mathcal{G}_0} + br_{J/\psi \rightarrow \gamma^* \rightarrow \phi\mathcal{G}_0} = 6.5 \times 10^{-4} (\alpha_s/0.33)^2 \quad (10)$$

in J/ψ decays.

These results implicitly arise because the traditional three-gluon exchange process is dominant over the EM one in J/ψ decays. Also, it provides a way to estimate glueball production in J/ψ decays, which can be normalized by hadron-hadron final states.

However, inspecting the gluon exchange process, we note the possible existence of a lower order diagram for $J/\psi \rightarrow \phi\mathcal{G}_0$, which could further enhance the glueball production branching ratio. In Fig. 3, we show that if the glueball is produced with one gluon directly from the $c\bar{c}$ annihilation, its coupling will be $O(1/\alpha_s)$ bigger than the mechanism of Fig. 1 (c) assuming all the gluons are perturbative. We thus estimate the contribution of Fig. 3:

$$br_{J/\psi \rightarrow 3g \rightarrow \phi\mathcal{G}_0}^{add} \simeq \frac{1}{\alpha_s^2} br_{J/\psi \rightarrow 3g \rightarrow \phi\mathcal{G}_0} \simeq 6.0 \times 10^{-3}. \quad (11)$$

We note that the cancellation of the strong coupling constant does not mean that $br_{J/\psi \rightarrow 3g \rightarrow \phi\mathcal{G}_0}^{add}$ is independent of α_s . The estimate of Eq. (6) should have contained strong coupling α_s in the experimental value for $br_{J/\psi \rightarrow 3g}/br_{J/\psi \rightarrow \gamma^*}$. With $\alpha_s \sim 0.33$ this enhances the branching ratio to be

$$\bar{br}_{J/\psi \rightarrow \phi\mathcal{G}_0}^{th} = br_{J/\psi \rightarrow \phi\mathcal{G}_0}^{th} + br_{J/\psi \rightarrow 3g \rightarrow \phi\mathcal{G}_0}^{add} \simeq 6.6 \times 10^{-3}. \quad (12)$$

It shows that if all the gluons are perturbative, glueball production would be strongly favored in J/ψ decays and suggests that large glueball production ratios can be driven by the dominant process of Fig. 3. However, cautions should be given to any over-interpretation of this estimate. We note that the validity of Fig. 3 dominance will strongly depend on the exchanged gluons being perturbative, which is not well satisfied as in the case of heavy quark production. While the actual numbers therefore may be debatable, the broad conclusion following from the Brodsky *et al.* approach seem robust.

In summary, the ideas of Ref. [1] may apply to $J/\psi \rightarrow \phi\eta'$ and in turn to glueball production. Compared to the subprocess $J/\psi \rightarrow \gamma^* \rightarrow \phi\eta'$, scalar glueball production via $J/\psi \rightarrow \gamma^* \rightarrow \phi\mathcal{G}_0$ is found to be the same order, which is consistent with the pQCD calculation by Kroll and Passek-Kumerčki [12]. However, for glueball production, it seems likely that a possible contribution from a lower order diagram may be dominant over the mechanism of Ref. [1] and the conventional three-gluon exchange process. Therefore, we advocate searching for the manifestation of glueball degrees of freedom in exclusive processes, $e^+e^- \rightarrow J/\psi \rightarrow \phi\mathcal{G}_0$, which can be underpinned by the experiments from the J/ψ factories (BES III, CLEO-c).

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 - [13] Indeed, the $\phi\mathcal{G}_0$ prediction will refer in practice to a mixture of these states. Ultimately, the relative production of these scalars may help to determine their relative \mathcal{G}_0 contents.

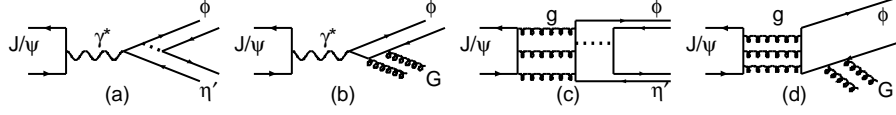


FIG. 1: Feynman diagrams for $J/\psi \rightarrow \phi\eta'$ and $J/\psi \rightarrow \phi G_0$ via virtual photon (a)-(b) and three gluon exchanges (c)-(d).

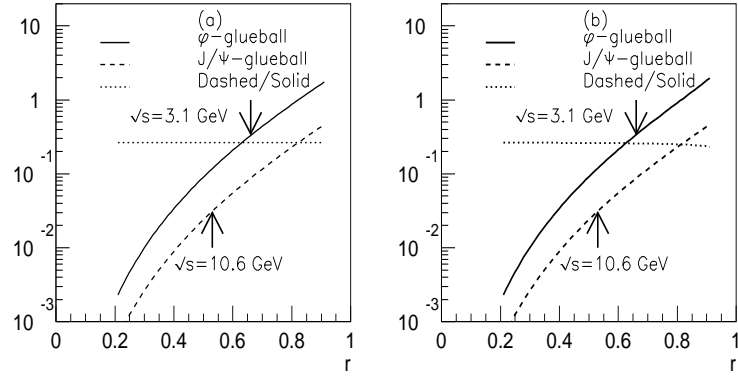


FIG. 2: Branching ratio fractions multiplied by 10^4 for ϕ -glueball (solid) and J/ψ -glueball (dashed) production via virtual photons, respectively. The dotted curve is the ratio of the solid to the dashed, of which the stable value shows the validity of rescaling the kinematics. The arrows denote the locations of r corresponding to the c.m. energies of $\sqrt{s} = 3.1$ GeV and 10.6 GeV for these two reactions.

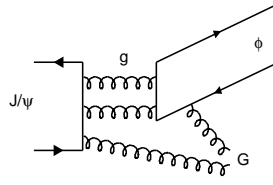


FIG. 3: Diagram for $J/\psi \rightarrow \phi G_0$ via lower-order gluon exchanges.